Cost and Benefits of Job Security Programs *

Sebastian Flennerhag†, Matteo Zambanini‡

June 2015

Abstract

A ubiquitous element of labor market institutions in developed countries are their many policies geared towards greater job security. This need arose during the deplorable condition of workers during the industrial revolution and the periods after it and were almost certainly warranted. Today, however, the rationale for most of these rules are taken for granted despite the fact that their effects on modern economies have rarely been studied. In this paper we aim to evaluate how labor protection policies affect macro-economic variables in a closed economy. We assess whether these policy regimes are effective given their intended outcome and whether their full effects generate a welfare enhancing economic environment. Our research will be focused on two main measures which affect respectively rigidity in the labor market and rigidity on the level of wages. We use an Infinitely Lived agents framework with perfectly competitive markets, and find that while the policies achieve their directly intended outcome, they are on the aggregate not welfare enhancing, although the difference is slight and some caution should be exercise.

Keywords: Job Security, Wage Rigidity

*Term paper, 5309 Dynamic Macroeconomic Analysis, Stockholm School of Economics

†22273

‡92844

Contents

Introduction

Labour law have always been a central theme in the political and economical policy in all developed countries. Many politicians found their programs on these themes in order to raise consensus, as these acts have a easy-to-understand and direct effect on the life of everyone. Labor law started to gain importance during the first industrial revolution, back in the 19th century, when a new class of workers moved from the countryside to the city in the hope of higher employment opportunities and better working conditions. However, deplorable conditions, long and exhausting working hours, low salaries, child labour and high risk of injuries or death forced governments to take action and step in to protect the weak laboring class. Starting with the British "Health and Morals of Apprentices Act" in 1802, different kind of reforms were adopted by all governments to help better conditions for workers. Nowadays, labour law consists in a collection of laws which regulate all the possible aspects of a working contract, from regulation of the relationship between the individual employee and the employer, to regulation concerning trading unions.
This paper focuses on policies that are aimed at the individual's employment terms and minimum wage, while abstracting from policies on living wages, working hours, health and safety conditions as well as discrimination laws etc.

Job Security

The main objective of this kind of regulation is to prevent employers from firing employees without any legitimate motive. Even if it is desirable to secure minimum levels of employment stability and transparency, this branch of labor law has been used in many developed countries as a macroeconomic tool to adopt counter-cyclical measures and maintain the unemployment rate under a desired level. In Europe, for instance, it is common to find very articulated requirements for firms when it comes to fire people and in most cases, when the number of employees dismissed reaches certain thresholds, additional rights and veto power are given to trade unions and governments. Even if these measures ensure stability to individuals, it has been argued that this rigid system makes the labor factor much more expensive to firms which will make investment decisions taking the increased cost into account. A question is then whether these costs are unduly high or whether the added security to household outweighs these costs. Moreover, another critique against this policy relates the average unemployment period of an individual who has lost his job or has just entered the job market. As the ability to make employees redundant decreases, the risk of employment increases and so a consequence of such policies can be reduced hiring, which in turn translates into longer time in unemployment for those that are made redundant. In addition, as a consequence of potentially longer unemployment periods, many currently employed workers are disincentivized from leaving their current position to find a new job.

Finally, a cause for concern relates the fact that longer unemployment periods reduce the human capital accumulation of individuals who will be less productive as a consequence.

Minimum Wages

The intuitive appeal of these types of policies is fairly straight-forward; its prevents employers from exploiting a potential power asymmetry, and safe-guards employees from volatility in what is typically their primary income source. While this policy can be beneficial from the employees point of view, the effects on firm's investment decision is not so clear cut and indeed it would seem that such a policy is rather detrimental. For instance, during a downturn in the economy, the demand for factor inputs such as labor is reduced which according to standard economic theories of supply and demand should result in lowered factor price, i.e. wages. However, with a minimum wage this may be impossible, and consequently the price on labor is kept artificially high despite the lower demand, lowering it even further. This suggest that this policy may be one with a populist undertone, it benefits the majority (the workforce) at the minority's expense (those employed in the margin). Presently, we shall investigate both these policy regimes formally.

Model

Presentation

We develop a one-sector general equilibrium model with endogenous labor and infinitely lived agents. Households maximize their expected utility over time taking prices as given, subject to their budget constraint. Their budget require them to balance their income sources with consumption and savings.

$$\begin{aligned} \underset{\{c_t,\ell_t\}_{t=0}^{\infty}}{\text{maximize}} \quad & E_0 U = \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \right\} \\ \text{subject to} \quad & c_t = (1+\rho_t)a_t + w_t\ell_t - a_{t+1}. \\ & a(0) = a_0 \quad \text{given.} \end{aligned} \tag{I}$$

Where $u(\cdot,\cdot)$ is a utility function that is strictly concave in both input arguments and satisfy the Inada conditions. a denotes assets while w_t and ρ_t are the supply-side factor prices that household face. c_t and ℓ_t denotes consumption and labor choices respectively and β is the time-preference discount factor. For each period, there is a perfectly competitive production market, where each firm has an identical technology. There is a continuum of firms indexed on the interval $[0, 1]$ such that they have a unit mass. Each firm $\omega \in [0, 1]$ take prices as given and optimize their current-period profits;

$$\begin{aligned} \underset{\{\ell_t^\omega, k_t^\omega\}}{\text{maximize}} \quad & \max\{A_t f(k_t^\omega, \ell_t^\omega) - w_t \ell_t^\omega - r_t k_t^\omega, 0\} \\ \text{subject to} \quad & \\ & \ell_t^\omega \in [\tilde{\ell}_{t-1}^\omega, 1] \\ & A(t) = A_t \quad \text{given.} \end{aligned} \tag{II}$$

Hence, firms choose to produce if their profits are at least 0. At each point in time they face a lower bound $\tilde{\ell}_t^\omega$ on the amount of labor they can hire. Typically, this is set to 0. Later, we will define a policy that determines this lower bound as a function of the labor choice in the previous period. The functions involved are $f(\cdot,\cdot)$ which is some production technology that is homogeneous of degree 1, increasing and strictly concave in all inputs and satisfy the Inada conditions. Note that the price of the good does not feature in this model, as we assume perfect competition and normalize the price to 1. Similar to above, w_t, r_t are the demand side-factor prices. Note that i) the capital of the firm is as of yet not identical to the consumer's assets and ii) the price on asset and capital are two different parameters. We shall use these facts to establish the no-arbitrage condition from which we can retrieve Solow's law of motion for capital. Lastly A_t is a stochastic variable that follows a Markov Chain. We let $\mathcal{A}$ denote the set of permissible states of technology, with n such discrete states;

$$A_t \in \{A_1, A_2, ..., A_n\} = \mathcal{A}.$$

We shall frequently use the notation $j \in \mathcal{A}$ as a short-hand for $A_j \in \mathcal{A}$. Furthermore, we define a Markov Chain in the usual manner and construct its associated matrix. First, the first order probability condition is

$$P(A_{t+1} = A_j | A_0, A_1, ...A_t) = P(A_{t+1} = A_j | A_t).$$

Given these, the time t conditional probability distribution entails

$$\sum_{j \in \mathcal{A}} p_{i,j} = 1, \ i \in \mathcal{A}.$$

We summarize this information in the transition matrix,

$$\begin{aligned} \Theta &= [p_{i,j}], \\ P(A_t) &\in \prod^t \Theta = \Theta^t. \end{aligned} \tag{1}$$

To tie the production and consumption side together, we impose aggregation rules such that the market prices clears the supply and demand side of factor input;

$$
\begin{aligned}
&\text{No Arbitrage Condition:} && \rho_t = r_t - \delta \\
&\text{Total Output Aggregation:} && \int_0^1 A_t f(k_t^\omega, \ell_t^\omega) d\omega = A_t F(K_t, L_t) \\
&\text{Asset and Capital Aggregation:} && \int_0^1 a_t^j dj = K_t \\
&\text{Labor Aggregation:} && \int_0^1 \ell_t^j dj = L_t \\
&\text{Feasibility Constraint:} && \int_0^1 (c_{j,t} + i_t^j) dj \leq A_t F(K_t, L_t).
\end{aligned}
\tag{III}
$$

Where i_t^j denotes the investments of individual j, implicitly defined through the budget constraint in (I). We include it here for completeness sake and make use of it to retrieve the LOM of capital. Finally, the functional forms we use are the familiar

$$f(k_t, \ell_t) \equiv k_t^\alpha \ell_t^{1-\alpha}, \qquad \alpha \in (0,1) \tag{F.1}$$

$$u(c_t, \ell_t) \equiv \log(c_t) + \theta \log(1 - \ell_t), \qquad \theta \in (0,1). \tag{F.2}$$

Solving the Model

In this section, we study the conditions under which an equilibrium exists and we show that the lower bound on capital does change the demand of resources, but the zero profit condition of the unconstrained firm remains. Consequently, production will is feasible under the constraint, and we show that with an auxiliary wage restriction the decentralized economy is isomorphic to the Social Planners Problem.

The Firm's problem

First, let us introduce a shadow firm that is unconstrained in its maximization;

$$\underset{\ell_t^\omega, k_t^\omega}{\text{maximize}} \quad A_t f(k_t^\omega, \ell_t^\omega) - w_t \ell_t^\omega - r_t k_t^\omega.$$

Given the Inada conditions, the solution is in the interior with the usual FOC;

$$A_t f_k' = r_t$$

$$A_t f_\ell' = w_t$$

Given these, under the aggregation rules of (III), the decentralized economy coincide with the SPP. Let the equilibrium choice of capital and labor from this problem be $(\bar{k}_t, h_t)$. Our original problem, (II), is a static nonlinear optimization problem with a strictly concave goal-function and a linear inequality constraint. The Karusch-Kuhn-Tucker FOC are both necessary and sufficient. We form the Lagrangian[1] and derive FOCs;

$$\max_{k_t^\omega, \ell_t^\omega} \mathcal{L} = A_t f(k_t^\omega, \ell_t^\omega) - w_t \ell_t^\omega - r_t k_t^\omega - \mu(\tilde{\ell}_t^\omega - \ell_t^\omega)$$

$$\mathcal{L}'_k = A_t f'_k - r_t = 0$$

$$\Rightarrow A_t f'_k = r_t \tag{2}$$

$$\mathcal{L}'_\ell = A_t f'_\ell - w_t + \mu = 0$$

$$\Rightarrow A_t f'_\ell = w_t - \mu \tag{3}$$

$$\mu \geq 0,\ \tilde{\ell}_{t-1}^\omega - \ell_t^\omega \geq 0,\ \mu(\tilde{\ell}_{t-1}^\omega - \ell_t^\omega) = 0.$$

Note that (2) is the usual marginal productivity of production. (3) however give rise to a non-standard system than requires some attention. Suppose the unconstrained problem yields $h_t > \tilde{\ell}_t$. Then, complimentary slackness imply $\mu = 0$ and we retrieve the standard marginal productivity equation. Note that, at these levels, optimal input choices yield zero profits since the production function is homogeneous of degree 1;

$$A_t f(k_t, \ell_t) = f'_k k_t + f'_\ell \ell_t = r_t k_t + w_t \ell_t.$$

By Euler's Homogeneous Function Theorem. Thus, a firm is indifferent between entering and not and with uncountably infinite number of firms, the mass of entrants is again infinite and indexed over $[0, 1]$. However, suppose instead that $h_t < \tilde{\ell}_t$. In this case, the constraint is binding and holding capital fixed at $\bar{k}_t$, (3) imply that

$$A_t f'_{\tilde{\ell}} < w_t \Rightarrow \mu > 0.$$

However, (2) is still a condition of an optimal point, and at $\tilde{\ell}_t$ this is not satisfied for the control $\bar{k}_t$, as this choice was optimal for h_t. Now, since $f(\cdot, \cdot)$ is strictly concave, $f'_k(\bar{k}_t, \tilde{\ell}_t) > f'_k(\bar{k}_t, h_t)$. This follows from the fact that the Hessian matrix must be negative definite for a strictly concave function and since$f^2_{k,k} < 0$, we must have $f^2_{k,\ell} > 0$. Therefore, we find;

$$A_t f'_k(\bar{k}_t, \tilde{\ell}_t) > r_t.$$

[1]The Inada conditions on $f(\cdot, \cdot)$ and $u(\cdot, \cdot)$ exclude corner solutions of the form $\ell_t = 0, \ell_t = 1, k_t = 0$.

From this fact we can deduce the optimal constrained capital choice, since $f^2_{k,k} < 0$, the firm responds by *increasing* capital, which in turn means that the marginal rate of productivity of labor increases. This is a crucial feature of the model for an equilibrium to exists. To illustrate the importance of this question, note from the profit function that (3) and (2) imply

$$\begin{aligned}\pi_t &= A_t f(k_t, \tilde{\ell}_t) - r_t k_t - w_t \tilde{\ell}_t \\ &= A_t f(k_t, \tilde{\ell}_t) - A_t f'_k k - (A_t f'_{\tilde{\ell}} + \mu)\tilde{\ell}_t = 0 - \mu \tilde{\ell}_t.\end{aligned} \tag{4}$$

By Euler's Homogeneous Function Theorem. Compare this result to the original problem in (II), unless $\mu = 0$ there will be no production in the economy and so the economy is undefined. However, we noted that capital increases as a response to a constrained choice of labor, which in turn increases $A_t f'_{\tilde{\ell}}$, implying that μ goes to 0. Formally, by Euler's theorem, the derivative of a k-degree homogeneous function is itself homogeneous by degree $k-1$. In our case, this means that f'_i is homogeneous of degree 0. Now consider the unconstrained optimal solution, where $A_t f'_k = r_t$. Let v be the fraction $\tilde{\ell}/h_t$ of additional labor required in the constrained problem. Then, for (2) to hold, capital must increase *by the same fraction*;

$$r_t = A_t f'_k(\bar{k}_t, h_t) = A_t f'_k(v\bar{k}_t, vh_t) = A_t f'_k(k_t, \tilde{\ell}). \tag{5}$$

Where the first equality stems from the unconstrained FOC, the second uses Euler's theorem, the third uses the definition of v. Altogether we find that the optimal constrained choice of capital is $k_t = v\bar{k}_t$, that is, an increase by the same fraction. Equation (5) establishes that the increase in capital is proportional to the policy-induced increase in labor. It then follows;

$$A_t f'_{\tilde{\ell}}(k_t, \tilde{\ell}_t) = A_t f'_{\tilde{\ell}}(v\bar{k}_t, \tilde{\ell}_t) = A_t f'_{\ell}(v\bar{k}_t, vh_t) = A_t f'_{\ell}(\bar{k}_t, h_t) = w_t. \tag{6}$$

Note that this result implies $\mu = 0$, and hence the profit function in (4) equal 0. Thus, by (5) and (6) the solution to the constrained maximization problem is structurally symmetric to the unconstrained solution. In particular, the solution to (II) is given by

$$A_t f'_k = r_t \tag{2'}$$

$$A_t f'_{\ell} = w_t. \tag{3'}$$

Note that these are the same conditions as under an unconstrained economy. We conclude that the firm's problem in (II) has an identical solution to its unconstrained counterpart, albeit with different factor inputs. Since the base-line infinite-horizon model is isometric to the Social Planner's Problem, this must also be the case for the model in (I)-(III). Invoking the aggregation rules in (III), we find the representative firm; all firms behave identically and the production market has a unit measure. Thus the product market is characterized by a stand-in firm.

The Household's problem

In what follows, standard results will be offered without derivation, in order to avoid an unnecessarily dense discussion. We refer the reader to Appendix A for derivations of value functions. Beginning in (I), we note that (III) implies that $a_t = k_t$, and that $\ell_t^\omega = \ell_t$. This implies that we can rewrite the budget constraint in (I) as[2]

$$
\begin{aligned}
a_{t+1} &= (1+\rho_t)a_t + w_t\ell_t - c_t \\
&\Leftrightarrow \\
k_{t+1} &= (1-\delta)k_t + r_t k_t + w_t\ell_t - c_t. \qquad (7)
\end{aligned}
$$

Now, before implementing this equation as a constraint on the consumer, we must note crucially that in so doing we are importing the constraint of the firm, as that is the relevant constraint for k_t and ℓ_t^ω. Now, the implication would be that not only are firms restricted in their firing policy, households are restricted in their ability to voluntarily quit. This naturally, would be a rather deplorable consequence of the model and so we must constrain the economy in such a way as to avoid it. Now, providing job security hinges crucially on households actually wanting to stay in employment, and so two natural ways to incorporate this feature are i) to make the constraint on the firm conditional on the optimal choice of the consumer or ii) set the wage such that the constrained labor choice of the firm coincide with a voluntary supply of labor by the households. In this paper, we shall opt for ii) for two reasons. Firstly, condition i) implies that the firm's problem and the consumer's must be solved independently which complicates things considerably. Secondly, in reality the policy safeguards against discrete drops in labor, from full employment to nil. Typically, a person on the margin of employment would very strongly like to stay in employment rather than forced exit. Given that we use a stand-in household and interpret the labor choice as a fraction of population in employment, this strong desire is omitted as the labor function takes a much smoother shape. Consequently it is reasonable to model the economy such that labor should be kept high, but the wage set in such that the supply of labor is a voluntary choice from the point of view of the household. While a more complex model might be able to better capture these features, we argue that our approach has a natural interpretation and serves well as a first approach to the subject matter. Now, let us study how the household in (I) behaves taking prices as given. Substituting the constraint into

[2]Note that the production feasibility constraint in (III) together with $a = k$ yields the LOM of capital:

$$
\begin{aligned}
k_{t+1} &= c_t + (1-\delta)k_t - r_t k_t - w_t\ell_t \\
&= [f(k_t,\ell_t) - i_t] + (1-\delta)k_t - r_t k_t - w_t\ell_t \\
&= [r_t k_t + w_t\ell_t - i_t] + (1-\delta)k_t - r_t k_t - w_t\ell_t \\
&= (1-\delta)k_t + i_t.
\end{aligned}
$$

the utility function gives

$$u(a_t, a_{t+1}, \ell_t),$$

From which the associated value function is

$$V(a_t, A_i) = \max_{a_{t+1}, \ell_t} \left\{ u(a_t, a_{t+1}, \ell_t, A_i) + \beta \left(\sum_{j \in \mathcal{A}} p_{i,j} V(a_{t+1}, A_j) \right) \right\}.$$

The first order condition with respect to labor is

$$\frac{\partial V}{\partial \ell} = u'(a_t, a_{t+1}, \ell_t) = 0.$$

This equation pins down the optimal choice of labor, given prices and the two-period asset path. However, the above equation is also an implicit function for the wage required given some asset path and labor choice. Now, suppose the firm is constrained to keep $\tilde{\ell}_t$ in the payroll. We can then solve for the wage that makes $\tilde{\ell}_t$ the unconstrained optimal choice of labor for the consumer. To retrieve the wage it is necessary to impose some functional forms. Hence, substituting for $u(\cdot, \cdot)$ from (F.2) we obtain;

$$\frac{w_t}{(1+\rho_t)a_t + w_t\tilde{\ell}_t - a_{t+1}} - \frac{\theta}{1-\tilde{\ell}_t} = 0.$$

Rearranging for wage gives

$$w_t = \theta \frac{(1+\rho_t)a_t - a_{t+1}}{1 - \tilde{\ell}_t(1-\theta)}.$$

Note that the above equation is increasing in $\tilde{\ell}_t$ as $\tilde{\ell}_t, \theta \in (0, 1)$. Consequently, in the case the firm-wise demand for labor is higher, so would the required wage be. Hence our minimum wage condition becomes,

$$w_t \geq \theta \frac{(1+\rho_t)a_t - a_{t+1}}{1 - \tilde{\ell}_t(1-\theta)}.$$

Finally, imposing the market clearing conditions in (III) gives the general equilibrium constraint as

$$w_t \geq \theta \frac{(1-\delta)k_t + r_t k_t - k_{t+1}}{1 - \tilde{\ell}_t(1-\theta)} \equiv \omega(k_t, k_{t+1}). \tag{M}$$

For brevity, we define $\omega(\cdot, \cdot)$ as the right hand side of (M). Let us pause here and study the consequences of this constraint. From the firms problem we know that a constrain labor choice increases demand for capital. Now, in a general equilibrium, the capital at time t is actually through households investment in previous period. Consequently, if the constraint is binding, the only variable in (M) is k_{t+1}, which increases with $\tilde{\ell}_t$. This has a very intuitive appeal; we saw from the firm's problem that the constraint induced increased demand for capital, which if capital remains fixed or adjust only marginally means that rental rates increase. This in turn would make it more attractive to hold capital, which in turn induce greater savings.

Finally, by the solution to the firm's problem as captured in (2′)-(3′) we can substitute in (7) in the households problem. Under (M) this results in an economy where the firm is potentially constrained in their choice of labor while the household is not. Since consumer's have identical preferences and initial assets, having a unit mass of households imply that the mass of household reduce to a stand in one. Through the isomorphism of the baseline model with the SPP, the general equilibrium of the decentralized economy in (I)-(III) is characterized by

$$\begin{aligned}
&\underset{\{k_{t+1},\ell_t\}_{t=0}^{\infty}}{\text{maximize}} \quad E_0U = E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t,\ell_t)\right\}\\
&\text{subject to}\\
&\qquad c_t = (1-\delta)k_t + r_tk_t + w_t\ell_t - k_{t+1}\\
&\qquad w_t = A_tf'_\ell \geq \omega(k_t,k_{t+1}|\tilde{\ell}_t)\\
&\qquad r_t = A_tf'_k\\
&\qquad \ell_t \in [\tilde{\ell}_t,1] = \mathbb{L}_t\\
&\qquad A_t \in \mathcal{A}\\
&\qquad P(A_{t+1}) = \Theta^t\\
&\qquad k(0)=k_0, h(0)=h_0, A(0)=A_0 \text{ given.}
\end{aligned} \tag{8}$$

Note that this is a problem with two control variables, k_{t+1}, ℓ_t, and three state variables, k_t and ℓ_{t-1} and the stochastic A_t. By substituting to constraints into the goal function, noting that (M) cannot be substituted for, the sequence problem of (8) is:

$$\begin{aligned}
&\underset{\substack{\{k_{t+1}\}_{t=0}^{\infty}\in\mathbb{R}^{\infty}\\ \{\ell_t\}_{t=0}^{\infty}\in\mathbb{L}_t^{\infty}}}{\text{maximize}} \quad E_0U = E_0\left\{\sum_{t=0}^{\infty}\beta^t u\left(k_t,k_{t+1},\ell_t,A_t|A_{t-1},\Theta\right)\right\}\\
&\text{subject to}\\
&\qquad A_tf'_\ell \geq \omega(k_t,k_{t+1}|\tilde{\ell}_t).
\end{aligned} \tag{IV}$$

Using functional forms, we find the final value function as[3]

$$V(k_t, \tilde{\ell}_t, A_t)_{A_t \in \mathcal{A}} = \underset{\substack{\{k_{t+1}\}_{t=0}^{\infty} \in \mathbb{R}^{\infty} \\ \{\ell_t\}_{t=0}^{\infty} \in \mathbb{L}_t^{\infty}}}{\text{maximize}} \Bigg\{ \log\left((1-\delta)k_t + A_t k_t^{\alpha} \ell_t^{1-\alpha} - k_{t+1}\right) + \theta \log(1-\ell_t) + \beta \left(\sum_{j \in \mathcal{A}} p_{i,j} V(k_{t+1}, \tilde{\ell}_{t+1}, A_j) \right) \Bigg\} \qquad \text{V}$$

$$\text{subject to} \quad A_t f'_{\ell} \geq \omega(k_t, k_{t+1} | \tilde{\ell}_{t-1}).$$

(V) embodies the solution the our model and we solve it numerically using the Value Iteration Algorithm. This completes the characterization of the model, and for the detailed oriented reader, further notes on the Value Function, the Markov Chain and the Value Function Algorithm can be found in appendices A-C respectively.

Policies

Job Security

We formally model job security as a lower bound on the labor choice of firms that is a function of the labor supply in the previous period. This captures the idea that while firms are able to let employee's go, there is a rigidity to amount of firing firms undertake in a given period. In Sweden firms are typically required to reallocate employees to other departments rather than outright firing them, and usually firing someone requires either exceptionally poor performance or the closing of a division. Formally, we impose the following lower bound of labor constraint on firms;

$$\tilde{\ell}_t = \lambda \ell_{t-1}^{\omega}, \quad \lambda \in (0,1), \quad \forall t \qquad \text{(P.1)}$$

Consequently, the inequality constraints on firms in (II) becomes

$$\ell_t \geq \lambda \ell_{t-1}.$$

Wage Security

Additionally, we contrast this policy with a different policy regime under which the wage instead has a lower bound, but the amount of labor is variable. Here job security is more directly oriented towards the work-force, as those who remain in it during a low-turn earn a

[3]See Appendix A for a general derivation.

higher wage. However, as we shall see, the amount of labor under this policy is considerably lower than in an unimpeded general equilibrium. Formally, we relax the lower bound constraint on the firm, essentially converting (II) into its unconstrained counterpart. However, we impose a minimum wage by modifying the market clearing wage,

$$w_t \geq \tilde{w} \quad \forall t. \tag{P.2}$$

Now, recall that in a general equilibrium, (3′) implies that the full characterization of this regime is

$$w_t = A_t f'_\ell \geq \tilde{w}.$$

By setting $\bar{w}$ at the average wage, say, we capture the idea of a real downward rigidity. Here this rigidity takes the form of an absolute value in our model, which imply that the policy is intended to insulate households from low-level wages and also reduce the downward-variability of their earnings which at face value seems like an attractive feature.

Calibration

Parameter Values

The focus of this paper is on the effect of policy regimes when the economy is hit by stochastic shocks. As such, the two most important set of parameters to consider are the probability transition matrix, and the policy regime parameters. The remaining parameters are set at values that are typically found in the literature, where most have been borrowed from McCandlesss (2008) who in turn has used the estimations of Hansen (1985) for the underlying, or in his terminology, "deep" parameters. From McCandless (2008; p. 60) we set the annual parameter values. However, as we are interested in short-term shocks, we use geometric averages to convert the time-variant parameter values to quarterly data, as summarized in table (1). Now, for constructing a Markov Chain, we wish to relate our

Parameters	Annual	Quarterly
β	0.98	0.99
δ	0.1	0.025
α	0.35	0.35
θ	0.5	0.5

Table 1: Underlying Parameter Values

findings to empirical data to the extent possible. Consequently, rather than constructing a stochastic environment arbitrarily, we use Tauchen's method (Tauchen, 1986) to elicit the transition matrix. As a first pass, we initially limited the number of feasible states to 2, but we found that using Tauchen's method the estimated transition matrix was too persistent with too narrow technology states. As a consequence most of the stochastic features of the model disappeared. We therefore followed Tauchen who reports from his simulations that a higher number of states is a more judicious choice for an accurate estimation, especially with high persistence in the AR(1) process. Hence, at a considerable expense in computing time we followed this advice and increased the number of states to 9. As a brief overview of the concepts involved, the underlying technology sequence follows an AR(1) process. In particular, we let the process have a mean of 1 and a stochastic element that follows[4]

$$z_{t+1} = \gamma z_t + \epsilon_t, \quad \gamma \in (0,1), \quad \epsilon \sim WN(0, \sigma_\epsilon).$$

Hence the technology state at a given time is $A_t = 1 + z_t$. The parameters γ and σ_ϵ are estimated from productivity data, for instance by regressing a Cobb-Douglas production function on GDP. Due to time constraints, we will refrain from estimating this process ourselves, and instead implement the estimation results from McCandless (2008; pp. 90-110). Letting the mean of A equal 1, the above equation means that A_t is distributed around 1, and for a suitable choice of error distribution cannot go negative. To find an estimate for γ, the Cobb-Douglas production function in (F.1) is estimated on US quarterly data with the production function parameter values above. McCandless reports the γ-value estimated by Hansen (1985), as this is a rather straigthforward affair of estimating the log-version of the Cobb-Douglas production function. Hansen finds that $\gamma = 0.95$ and that the standard error for output (in log form) is 0.0176. Using these estimation, McCandless elicit the implied standard deviation of the error term in the expression above, given $\gamma = 0.95$. Doing so, he finds $\sigma_\epsilon = 0.0032$. Tauchen's method is essentially based on appropately partitioning the set of possible technology states over a probability distribution, and in a sense discretize the AR(1) sequence. Using the values above we implement Tauchen's method using Flodén's Matlab routine (Flodén, 2008). Table 2 reports the estimated technology states, and table 3 reports the estimated transition matrix.

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
0.969	0.977	0.985	0.992	1.000	1.008	1.015	1.023	1.031

Table 2: Technology States

[4]For further details see Tauchen (1986).

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
A_1	0.764	0.235	0.001	0.000	0.000	0.000	0.000	0.000	0.000
A_2	0.059	0.741	0.200	0.001	0.000	0.000	0.000	0.000	0.000
A_3	0.000	0.075	0.757	0.168	0.000	0.000	0.000	0.000	0.000
A_4	0.000	0.000	0.093	0.767	0.140	0.000	0.000	0.000	0.000
A_5	0.000	0.000	0.000	0.115	0.770	0.115	0.000	0.000	0.000
A_6	0.000	0.000	0.000	0.000	0.140	0.767	0.093	0.000	0.000
A_7	0.000	0.000	0.000	0.000	0.000	0.168	0.757	0.075	0.000
A_8	0.000	0.000	0.000	0.000	0.000	0.001	0.200	0.741	0.059
A_9	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.235	0.764

Table 3: Technology Transition Matrix

Value Function Iteration

In order to hone in on a suitable range for the Value Function Iteration algorithm, we need to define discrete grids for initial capital stocks and possible lower thresholds. In order to do so, we start by characterizing the steady state values of the economy given the mean technology parameter. Since the final result in (V) is similar to the model described in McCandless (2008, pp.63-64), we omit derivations of FOC as they are identical apart from the additional nonlinear constraint[5]. For the purpose of finding a focal point for the grid, we fix A_t at the mean of the technology process which is set to 1. We proceed in the ordinary fashion by taking derivatives of the value function with respect to labor and one-period-ahead capital. Note that labor at time t does not feature into the value function explicitly for $t+1$, hence we need not consider $V'_{\ell_t}(k_{t+1})$. On the other hand, k_{t+1} does feature in the equation and using the envelope theorem on $V(k_{t+1}, \ell_t, A_{t+1})$ we find the two sets of FOC, together with the wage constraint as;

i) $$\frac{(1-\alpha)A_t k_t^{\alpha}\ell_t^{-\alpha}}{(1-\delta)k_t + A_t k_t^{\alpha}\ell_t^{1-\alpha} - k_{t+1}} - \frac{\theta}{1-\ell_t} = 0$$

ii) $$-\frac{1}{(1-\delta)k_t + A_{t+1}k_t^{\alpha}\ell_t^{1-\alpha} - k_{t+1}} + \beta\frac{(1-\delta) + \alpha A_{t+1}k_{t+1}^{\alpha-1}\ell_{t+1}^{1-\alpha}}{(1-\delta)k_{t+1} + A_{t+1}k_{t+1}^{\alpha}\ell_{t+1}^{1-\alpha} - k_{t+2}} = 0$$

iii) $$(1-\alpha)A_t k_t^{\alpha}\ell_t^{-\alpha} - \omega(k_t, k_{t-1}) \geq 0.$$

Focusing on the unconstrained case, we set $\tilde{\ell}_t$ sufficiently low to satisfy the unconstrained solution to i)-ii). Imposing steady state-conditions (denoted by dropped subscripts) and

[5]Note in particular that while our policy does introduce an intertemporal link between ℓ_t and $\tilde{\ell}_{t+1}$, firm's does not take this link into account, and households are unconstrained. Thus this link is not part of the optimizing behavior of the economy.

solving ii) for k yields

$$k = \left[\frac{1-\beta(1-\delta)}{\alpha\beta}\right]^{\frac{1}{\alpha-1}} \ell. \tag{9}$$

Note that the bracketed expression multiplying ℓ is a constant made up entirely of parameters. Furthermore, since $\delta < 1$ and $\beta < 1$, it is also positive. To simplify notation, let B denote the bracketed expression, i.e. $B = [(1-\beta(1-\delta)/\alpha\beta]$. Substituting the above expression for k into i) and rearranging yields

$$\ell = \frac{1-\alpha}{(1-\delta B^{-1})\theta + (1-\alpha)}. \tag{10}$$

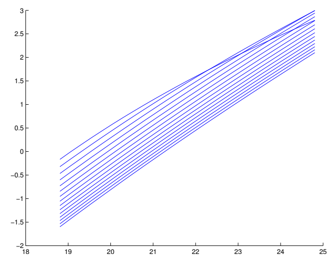
These equations for k and l can then be used to find steady state values for consumption, production and factor prices, reported in table 4. Given this information, we create a grid

k	ℓ	c	y	r	w
21.806	0.634	1.642	2.187	0.035	2.242

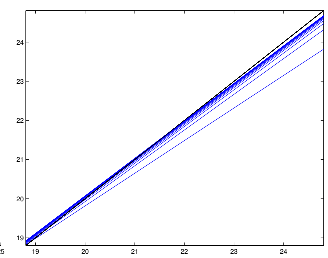
Table 4: Steady State Values of Endogenous Variables

for capital spanning $\{20, 24\}$ with equidistant nodes. Similarly, we use the steady state for labor to construct a grid. However, we find just as in McCandles's version of the basic Hansen-RBC model that the variance of labor is very low. Therefore, to get an accurate estimation, we use a rather narrow grid of potential lower states. We use equidistant nodes between $\{0.62, 0.65\}$, noting that the optimal unconstrained choice is never lower than 0.63 for our economy. Furthermore, to get accurate estimates to conduct a numerical analysis, we use 50 nodes for labor, while capital is not contained within a decimal range so we have a somewhat courser grid with 25 nodes. We will later discuss in detail the consequences for our analysis of this low variation. With respect to the minimum wage regime, we refrain from running it on the minimum labor grid. In general, these two policies are incompatible in the economy modeled here. To see this note that, since k_t is given, if labor is also constrained, w_t cannot change. Hence if $w_t < w$ the control space is an empty set. Consequently, the minimum wage regime is estimated using an identical procedure to the constrained labor regime, with the modification that the minimum labor grid is a scalar set to 0.1. Naturally, the nonlinear constraint is updated to that of (P.2). As we shall see, under a minimum wage regime labor frequently dips well below the levels in even the unconstrained case. Since all variables in this economy is real, one must ask to what extent a government can keep real wages at a minimum level, as they are primarily in a position to influence nominal wages. Here, we shall in general ignore such consideration due to time constraints, and instead allow the government to impose a minimum-wage slightly below the steady-state wage level from above. In particular we set $\tilde{w} = w - 0.04 = 2.02$. We find that this leaves the economy generally unconstrained, but forces firms to keep real wages high when the economy is running into a persistent downturn. Lastly, it should be mentioned that we could have replicated the the labor-constraint process for wages, which perhaps is a more realistic notion. Under such a regime, the government cannot set an exact threshold, but are instead in a position to impose some ”stickiness” on wages such that they decline more slowly. However, as a first pass we shall refrained from such due to time constraints and the computational costs of the labor-constrained regime. Indeed, running the VFI for the constrained labor regime with the above parameter values takes 22.5 hours over 96 iterations with a convergence tolerance of 0.01. Whether this level of precision is strictly needed may be debated, but the extremely narrow range of ℓ is a caution against too coarse a grid. In figure 1 we showcase the dynamics of the algorithm by plotting every sixth iteration where for the case with the lowest boundary on labor and $A = 1$.

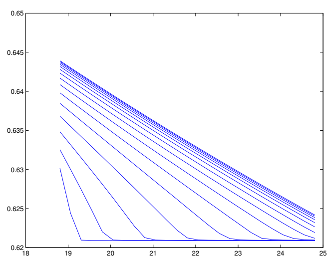
As can be seen in 1a, the value function is concave and converges over iterations. Similarly, panel 1b capital is increasing in initial value and crosses the 45°-degree line at about 21. It is interesting to study how the optimal policy of labor behaves, first when it is constrained (1c), and when the wage is constrained but labor unconstrained (1d). By comparing the movements in the optimal choice of labor with that of capital. As the optimal choice of capital increases, so does the choice of labor. Indeed, while the lower bound is binding for the first iterations, as capital increases the constraint is gradually less binding until it is irrelevant. Contrast this with a minimum wage; the effect is clearly to *reduce* the amount of labor in the economy sufficiently for the marginal productivity to equate the equilibrium wage rate. As the capital stock grows, the marginal productivity of labor increases allowing for more labor until the the capital stock is sufficiently large for the labor choice to be unconstrained. Clearly, this policy is of interest for those with secure employments and against those who is employed in the margin, as the policy induce volatility in employment.



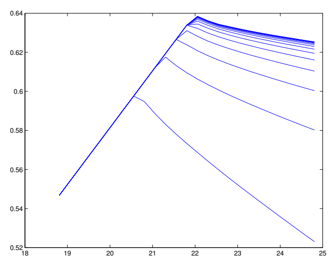
(a) Value Function



(b) Capital



(c) Labor with Minimum Lower Bound



(d) Labor w. Constrained Wage

Figure 1: The Value Function Iteration Algorithm

Evaluation

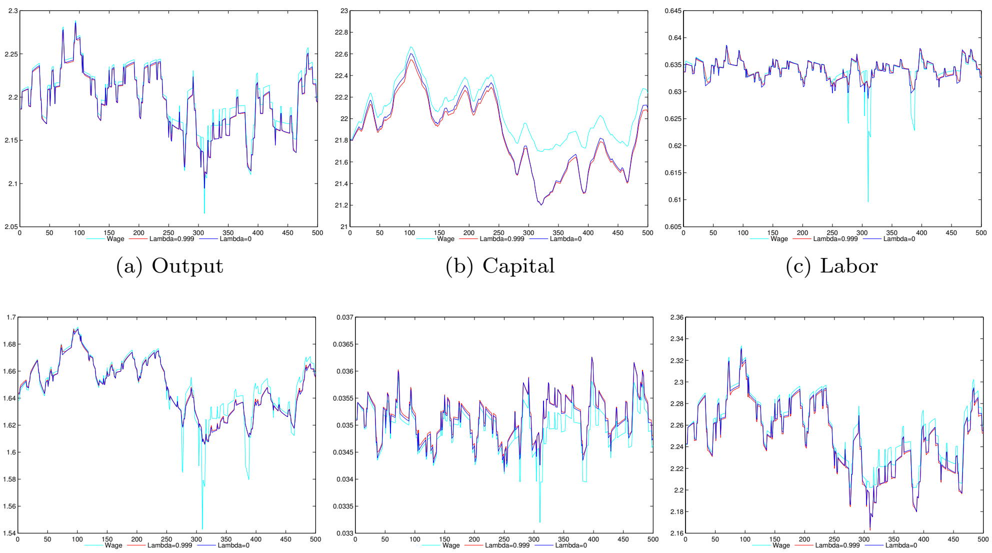
Simulation

Using the policy-functions from the VFI, we simulate an economy on a generated Markov Chain for each policy regime as well as an unconstrained economy, the results of which are displayed in figure 2. As can be seen from these figures, the simulated properties of the various regimes follow each other rather closely, with the fixed-wage regime being the most divergent regime. In particular, it would seem that the constraint on labor has a very limited impact on the economy. This, as it turns out, is an artifact from the generally low variance of labor, as can be seen in panel 2c. The unconstrained optimal choice of labor is bounded within a range as small as 0.01 implying that the economy is supplying labor almost in-elastically! Several reasons could explain this, here we offer a few. Labor influences utility directly through a multiplier that is larger than one, the wage, given the log-function of utility, small changes in labor can have very strong effects on utility, thus discouraging household's from inducing volatility in that variable. Similarly, small relative changes in capital have stronger level effects on consumption, so volatility in capital may be preferred to volatility in labor, here to the extent that labor is extremely stable. To better understand the short and long term dynamics of the different economies with and without the government interference, we will analyze briefly the mechanics and interrelation between variables in our model, in the three cases. We will start our analysis focusing on the first case, the wage-constraint policy.

We shall compare two economies, one with a wage constraints and one without, and assume they show the same variables' initial levels. As a negative shock happens, firms in the unconstrained case will see a reduction in their factors' productivity and they will immediately adjust their demand for these inputs reducing their amount and compensation (wages and interest rates drop). However, in the constrained economy, this reduction may be infeasible, leaving it with artificially high wage rate. Unable to put a downward pressure on wages, the artifically high cost causes a further decline in labor demand. This kind of behavior explains the sudden extreme dips in labor visible in panel 2c, as well as the generally higher wage in panel 2f. From an economic perspective, this policy has some stark consequences from a social conflict point of view; while those remaining in work enjoys a higher wage, many loose their employment at the expense of those who remain with an artificially high wage, in short the winner takes it all. However, wage rigidity has consequences also on the other macro variables in the economy, starting with the level of capital. In fact, as labor becomes a highly-costly input factor due to the minimum wage, demand for capital increases as it becomes relatively cheap. Obviously, the effect of lower labor and higher capital leads to a downward pressure on capital productivity and consequently on interest rates which drop significantly in relative terms. Another consequence of the higher level of capital in the economy can be found in the amount of labor and level of wages during good times. As expected, higher capital means higher productivity for the labor factor and thus higher compensation and higher employment rate during bullish periods.

A final interesting fact on capital is related to its volatility. It is clearly visible in 2b that the capital level in a wage-constrained economy stabilizes at a higher value and evolves with less significant swings. The reason behind this more stable path follows from how the interest rate reacts during a negative turn; as labor drops sharply, the interest rate increases, inducing higher investments during low turns which leaves the economy with more capital in steady state. Hence a positive shock causes a lower relative shock as the larger capital base need a lesser relative adjustment to facilitate the appropriate level change. Finally, we conclude the analysis of the first policy looking at the output evolution over time: as we can see, the production follows a path similar to the one of consumption, with higher values in good time compared to the unconstrained case (due to larger accumulation of assets and higher use of labor), and few sharp falls in bad times, following the trend in the labor market.

We will move now our attention on the second policy which limits the firms' ability to fire their current workforce. We will start looking at the level of labor as this is the variable directly influenced by the policy. As we can see, the evolution of this variable appears to be smoothed compared to the base case. The reason is simple: firms which want to reduce



(a) Output (b) Capital (c) Labor

(d) Consumption (e) Interest Rate (f) Wage

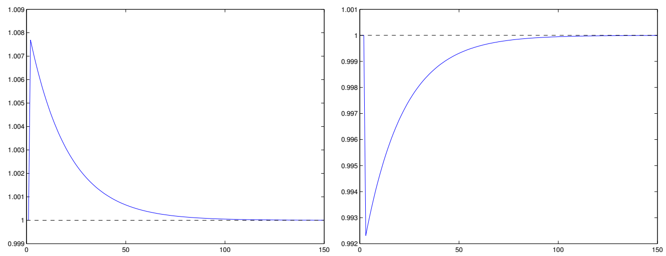
Figure 2: A Simulated Economy

the level of labor in their production during a recession, are only allowed to do it slowly. The slow and delayed reduction in labor has a direct consequence on the speed of capital reduction which in turn has a consequence on wages. All these three variables reduce progressively and stabilize at a higher level compared to the unconstrained case. In addition, as expected, this higher level for all variables has a positive effect on the production and consumption.

Predictable is also the behavior of the economy in this case when there is a positive technological shock. In fact, the higher level of capital leads to a steeper increase in labor, a higher demand for this input factor and to a higher level of compensation, which in turn translates into higher production and consumption in the short and long run. Even if this policy increases aggregated consumption in the economy, we have to remember that it requires agents to work more. We will come back to the discussion whether the policy is effectively welfare enhancing or not later in this paper.

Short Term Dynamics

Since the Markov Chain that generated the policy functions is a discretization of an AR(1) process for technology, we retrieve the original technology process and study how labor and capital react (in relative terms) when technology receives a one-off shock from its steady state at $A = A_5 = 1$ to its most probable transitions, being $A_{t+1} = A_6 = 1.008$ for a positive shock or $A_{t+1} = A_4 = 0.992$ for a negative shock. The results are depicted in



(a) Positive Shock (b) Negative Shock

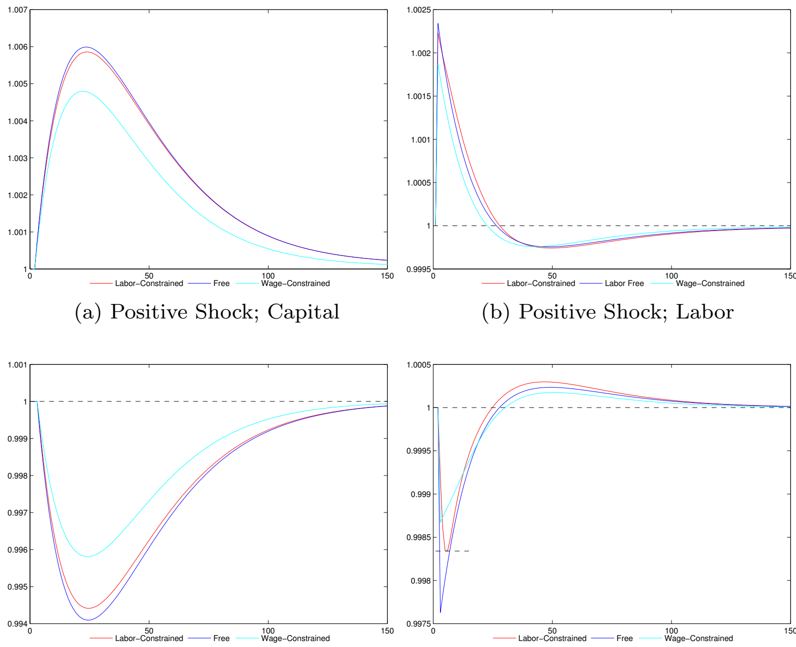
Figure 3: Shocks to Technology

figure 4. Several interesting features stand out. Beginning in the top panels, 4a and 4b, a positive shock increases the productivity of production inputs. Labor immediately shoots up while capital starts to be slowly accumulated. As new capital is used in the production, labor becomes more and more productive and less amount of work is needed. However, as technology comes back to its initial value both capital and labor slowly converge to their initial long term equilibrium. It is interesting to notice the small differences among the three economies: in the labor-constrained scenario, for example, labor seems to react less to the shock (in relative terms) and adjust slowly in the transition period. The reason is that in this economy the amount of labor used is higher than in the unconstrained case so, even if the magnitude of the swing is bigger, in relative terms labor reacts less. The same way of reasoning works also for capital in this scenario.

More trickier is the case of the wage-constrained economy which requires a a bit more attention. As we have previously said, due to the presence of a minimum wage level, all the downside wage risk is directly transferred to the labor so firms which have to pay too much for this production input will respond with a sharp drop in the demand for it. For this reason, we would expect extreme volatility but on the contrary, what we see is the opposite. In fact, labor seems to react less to a positive shock and adjust towards its long term mean faster than the other two cases. To explain this phenomenon we have first of all to clarify that, given the 'normal' initial state of the economy and the small technological shock considered, the wage-constraint is not binding. It is clear that in the extreme case, when wages should drop below the minimum level, labor swings are wide. Our attention, though, is now on the general case with a stable, normal economy. At this moment, we have to recall that in equilibrium this economy tends to accumulate more capital and use more work (since it is more productive). This initial situation explains why, given small technological shocks, smaller variation of the two inputs are required to reach a new equilibrium. The net result is less volatile variables.

An analogous case can be made for a negative shock. However, a new phenomena does arise. In particular, looking at figure 4d, we can clearly see the direct effects of the two

policies. In fact, as expected, labor takes longer to adjust to a negative shock in a labor-constrained economy. This delay leads to a lower level of labor adjustment and a faster process of adjustment. Interesting to notice that the turning point is very close to the point where the labor line crosses the labor line of the unconstrained economy (see dashed line). In this case the policy achieved its purpose: people are not fired by firms as the negative shocks happens, and as the economy comes back to its normal level the work force is already deployed, capital stock is maintained high and recovery is faster. For the wage-constraint policy, on the other hand, it is worth noting that the drop in labor with a negative shock is much lower than the increase with a positive one. The reason is again due to the initial capital level and its low volatility which keeps high the productivity of this input factor.
In conclusion, when studying relative effects of technology shocks, we see that labor-protection policies achieve their goal, preventing the drop in employment during negative periods from being too steep, stimulating a larger capital stock accumulation and hence maintaining a higher level of employment during the recovery. Likewise, since a positive shocks causes a temporary oversupply, the gradually re-settling keeps labor artificially high. A word of caution, the comparison made here is purely in relative terms from an equal-sized shock. As we shall shortly see, the effects in level-terms points to the fact that while the relative effects of the policy seems attractive, in level terms the conclusion may sway in the opposite direction.



(a) Positive Shock; Capital

(b) Positive Shock; Labor

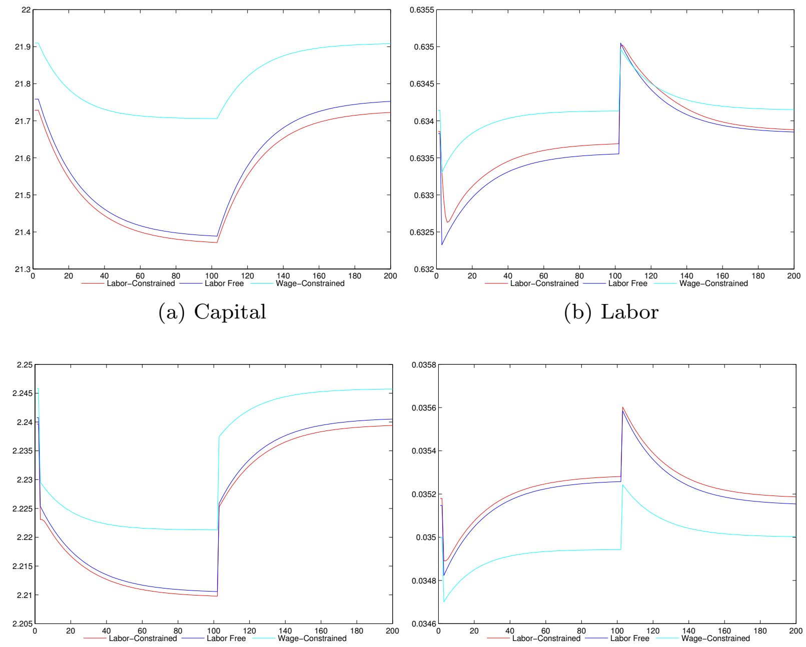
(c) Negative Shock; Capital

(d) Negative Shock; Labor

Figure 4: Impulse Response Functions

Long-Term Effects

We now turn to transitional dynamics. Rather than studying an impulse response, we instead ”permanently” shift the technology state from $A_5 = 1$ first to A_4 and then back to A_5, to study how the system transitions from one state to another. Here, we keep the variables in level terms to see aggregate effects on the economy. In figure 5 we can clearly see the relationship among the variables in the three economies. In panel 5c we see the slow reaction of labor in the labor-constraint policy which leads to an average higher level of the same variable, as well as a lower level of assets. As a direct consequence, wages are pushed down (due to higher work) and interest rates up (lower capital). On the opposite, as we have already seen, capital levels are higher in the wage-constrained scenario, leading to higher level of labor and wages, and lowering the average interest rate. A negative shock has a negative effect on labor but, given its relatively higher productivity, the effect is less pronounced than in the other two cases. As a final result, the policy seems to shift the relative mass of income sources towards labor, while reducing the per-unit compensation for holding assets. The policy seems to achieve its purpose being beneficial for the working force at the expense of capital holders, who consist in the wealthy part of the population. It is important to remember, though, that our model assumes all agents to be equal, with



(a) Capital

(b) Labor

(c) Wage

(d) Rent

Figure 5: Transitional Dynamics

same skills, same utility functions and same wealth. As a consequence, this kind of analysis would require a more sophisticated model than the one presented in this section. We will discuss the possible extensions which can be made to the basic model later in this paper.

Welfare Analysis

Up until this point, we have studied the general properties of the various regimes in a typical simulation, their short-term behavior, and long term transition properties. It is now time to tie the pieces together and study how the regimes impact the household's expected lifetime utility. To accomplish this, we let the economy adhere to the value function of (V) which gives rise to the expected utility

$$U^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t^*, \ell_t^*, A_t) \right\}.$$

Where c^*, ℓ^* denote optimal control given some initial technology state . Since U^* is non-linear, solving the expectancy operator requires a probability density function over the stochastic variable together with associated functional values. This is a rather intractable problem and we shall instead resort to numerical methods and approximate the expect utility through a Monte-Carlo simulation of technology sequences. Hence, given some policy regime, we compute (Note the change of horizon of the discounted utility)

$$U^* = \frac{\sum_{j=1}^{X} \sum_{t=0}^{T} \beta^t u[c_t^*(j), \ell_t^*(j), A_t(j)]}{X}. \tag{11}$$

Where we let j denote the jth simulation of a technology sequence. Again, the issue of low labor variation causes high computational costs. Since variance is so low, we need to use 3-dimensional interpolation methods which drives up the computation time for a given sequence. To retain some precision while keeping computational time down, we set the utility sequence to $T = 500$ and $X = 5000$. Running this simulation for the three regimes studied so far in this paper takes approximately 40 minutes. Table 5 reports the results. As expected, the wage-constraint policy which lead to higher volatility in both consumption and labor, is not welfare enhancing. It is interesting to note that the labor-constraint policy is neither welfare enhancing. As we previously saw, this policy lead to a constantly higher level of both consumption and labor and the net effect on the agents' utility was not straightforward. The result, as it turns out, is to the negative. Nevertheless, we should caution that the Monte-Carlo simulation conducted here is underpowered, and a longer simulation would be advisable. Further, our model studies only one agent, and perhaps a more realistic welfare analysi would require several types of agents.

Labor-Constrained	Unconstrained	Minimum Wage
-0.65873	-0.65868	-0.6657

Table 5: Expected Lifetime Utility

Extensions

Indivisible Labor

In this section, we discuss some potential extension of the baseline model presented so far as well as some tentative results. We shall focus on simple extensions that induce greater volatility in labor and in particular consider two alternatives; firstly, we implement Hansen's basic indivisible labor model, as presented in McCandless (2008, pp. 112-121), secondly, we consider an alternative policy-regime that does not directly constrain firms labor decision, but manipulates their profit function in order to reach similar results. With respect to the first, the modification is a straightforward one, rather than choosing its variable labor, households sign a contract with firms in period x to work h_x amounts in period t. A stochastic process determines if the household is one of those that has to work, but if they do not have to work a perfect unemployment insurance contract allow them a full wage payment nonetheless. As for now, we remain agnostic as to how this insurance contract is financed, and its implications for firm's profits. Since individuals are identical over a unit mass, we retain the stand-in household, and the representative households expected labor in period t denotes the fraction of the population called to work, we denote this by

$$h_t = \psi_t h_0.$$

The expected utility is

$$\begin{aligned} u(c_t, h_t) &= \log c_t + \psi_t \theta \log (1 - h_0) + (1 - \psi_t)\theta \log (1 - 0) \\ &= \log c_t + H h_t. \end{aligned}$$

Where $H = \theta \log (1 - h_0)/h_0$ is a constant. Note that we made use of $\log 1 = 0$ and multiplied the second term by h_0/h_0 to retrieve h_t. Now, since h_t is the amount of labor firms have, this is the relevant measure in the production function and budget constraint. Substituting ℓ with h_t, the firms problem is identical. As for the household, we note two facts. Firstly, since ψ is an exogenous parameter to the household, once h_0 is set they supply labor if called to work. Hence, at time t each household supply labor inelastically and so a minimum wage constraint is no longer a conceptually relevant condition[6]. Secondly, the new utility function means we must update the value function in (V). This gives the following value function

$$\begin{aligned} V(k_t, A_t)_{A_t \in \mathcal{A}} = \underset{\substack{\{k_{t+1}\}_{t=0}^{\infty} \in \mathbb{R}^{\infty} \\ \{h_t\}_{t=0}^{\infty} \in \mathbb{H}_t^{\infty}}}{\text{maximize}} \Bigg\{ & \log \left((1 - \delta) k_t + A_t k_t^{\alpha} h_t^{1-\alpha} - k_{t+1} \right) \\ & + H h_t + \beta \left(\sum_{j \in \mathcal{A}} p_{i,j} V(k_{t+1}, A_j) \right) \Bigg\}. \end{aligned} \tag{V'}$$

To be able to make comparisons across models, we wish to parametrize ψ_t and h_0 such that the steady state value of the two models equate. Now, the only difference between them is in the second sub-utility function. Consequently, the relevant difference will be with respect to the expression for steady state labor. As for this model, we find this expression by taking the derivative of the utility function with respect to labor.

$$\frac{w_t}{(1 - \delta)k_t + r_t k_t + w_t h_t - k_{t+1}} + H = 0.$$

[6]In general, this omission could cause the intertemporal dynamics of capital to change, as the minimum-wage constraint induced greater savings. However it is not clear what a comparable restriction would be in the model with indivisible labor. Furthermore, we are primarily interested in generating a more plausible volatility in labor. As such, we shall refrain from a more involved policy regime.

Imposing steady state conditions and substituting for k through (9) gives us h as

$$h = \frac{1-\alpha}{H(\delta B^{1/\alpha} - 1)}.$$

Since we wish to equate h with ℓ, we combine the above equation with the steady state expression of ℓ in (10);

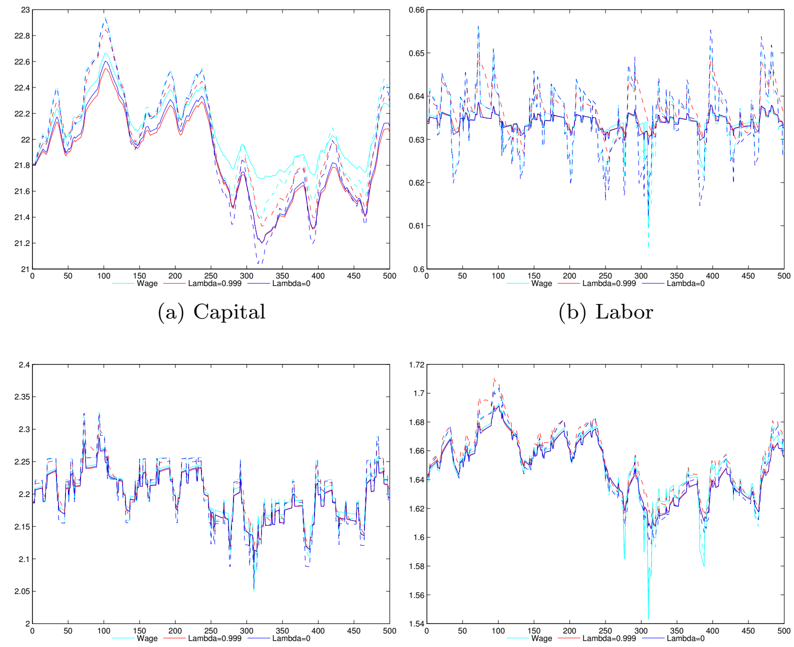
$$\frac{1-\alpha}{\delta B^{1/\alpha} - 1}\frac{1}{H} = \frac{1-\alpha}{(1-\delta B^{-1})\theta + (1-\alpha)}.$$

Note that H features in it. Rearranging for h_0 gives;

$$\frac{h_0}{\log(1-h_0)} = \theta\frac{\delta B^{1/\alpha} - 1}{(1-\delta B^{-1})\theta + (1-\alpha)}.$$

Since all terms on the right hand side are parameters, we can use numerical methods to find h_0, and estimate it to $h_0 = 0.9187$ from which we retrieve $\psi = 0.6900$ and $H = -1.3657$. Since we have chosen parameter values such that $h = \ell$, it follows that the steady state value of capital, and hence remaining variables, remain as in table 4. As seen in 6b, this model produce considerably more volatility in labor. As such, we get good approximation with a less dense grid. Therefore we halve the size of both grid and let the grid of initial capital be constituted by 13 nodes while potential lower bounds on labor is constituted by 25 nodes. The VFI was completed in 14 hours over 232 iterations[7]. The model was also estimated under a fixed wage regime in a similar manner to above, but not over the labor-constraint grid and instead subject to the wage constraint in (P.2). We shall now undertake a brief analysis of the differences of these models, with a primary interest in labor. In the following figures, dashed lines represent the indivisible labor model, while solid lines represent the original model. Figure 6 displays a simulation of both models under all regimes and the overall impression is clear: the indivisible labor model generates much more volatility, especially in labor. In particular, figure 6b hints at a much more efficient labor-constraint policy regime, as the level of labor employment is almost universally higher than under an unconstrained regime. This in turn has a material impact on capital; from 6a one sees that the capital difference between the two regimes is now material. Furthermore, the capital stock is lower during high periods, but higher during persistent lows and through the following upturns. As a consequence, these periods with persistent low states of technology sees materially higher output and consumption leaving the valuation of its welfare impact more ambiguous than before.

[7]Note that the number of iterations almost doubled, which was caused by the reuse of the same tolerance level. This was perhaps rather unnecessary. With divisible labor the value function is quite flat requiring much higher precision. The greater curvature of the value function of the divisible labor model could have been well approximated at a less strict tolerance level.



(a) Capital (b) Labor

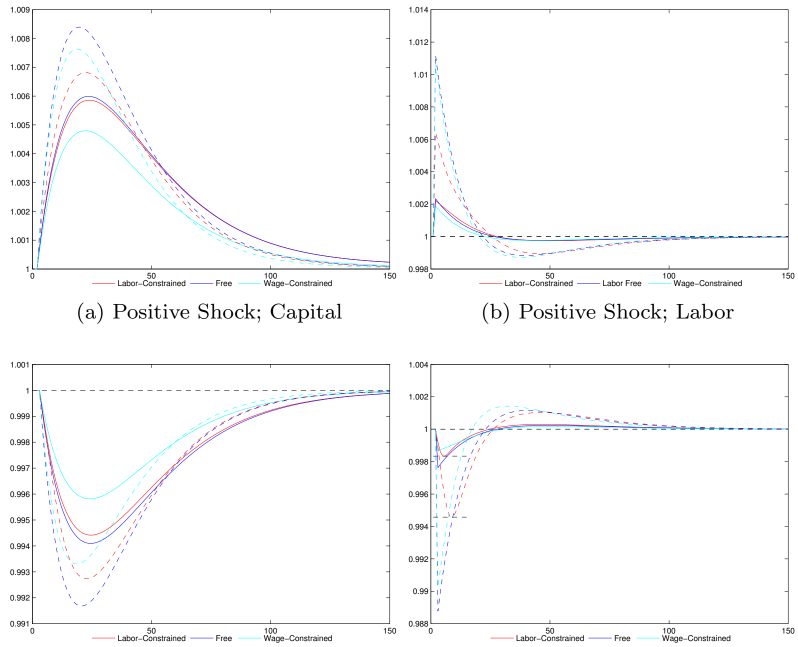
(c) Output (d) Consumption

Figure 6: Comparison of Simulated Economies

Looking now at the wage-constraint policy, we see that implementing this extension volatility increases for labor as well as for capital. It is interesting to notice that production's volatility increases as its underlying input factors become more volatile, but consumption becomes more stable. Having said that, consumption still remains much more volatile than in the unconstrained case.

Turning now to short-run shocks analysis, the effect of indivisible labor is clearly visible. Beginning with capital, a positive shock as in 7a seem to primarily produce a stronger reaction with indivisible labor while the temporal pattern is qualitatively unaltered, which is reassuring given that the minimum-wage constraint is not enforced in the indivisible labor model[8]. Intriguingly, while the timing of the maximum capital peak seems almost identical when the shock is positive, it is slightly temporally delayed when the constraint is binding, which is not the case under a divisible labor model. Not that if this was due to the minimum-wage constraint, the effect should have been the opposite, as it induces higher savings, but the model with the highest level of savings is the one without it. To further shed light on this difference we now turn to the effects of a technology shock on labor in the various models, as displayed in panels 7b and 7d. The first thing that jumps to our eyes is that the reaction

[8]Which hints at its irrelevance. In practice we have found in general be non-binding, i.e. the optimal savings choice is greater than what the constraint would have implied.



(a) Positive Shock; Capital

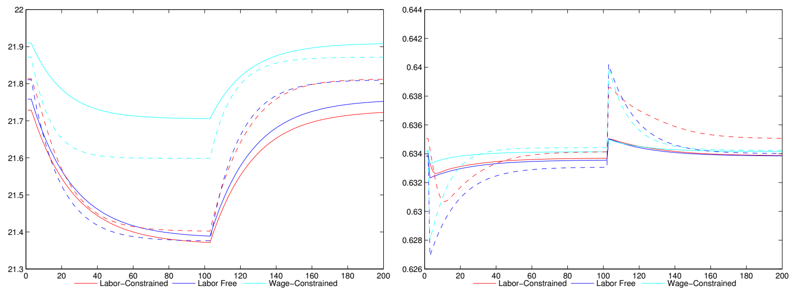
(b) Positive Shock; Labor

(c) Negative Shock; Capital

(d) Negative Shock; Labor

Figure 7: Comparison of Impulse Response Functions

of labor to this shock is much stronger than before. Second and more important aspect, the reaction patterns is no longer similar to the original model. Note in 7b that while the original model produces similar spikes with- and without the labor constraint, the spike with a labor constraint is now much lower. Furthermore, the delay is more protracted and while the capital stock is lower for the first 25 periods, it is instead higher in the interval $[25, 50]$ which is not an effect we originally had. It is interesting to notice that, even if the initial reaction is qualitatively identical (the turning point occurs when the unconstrained choice has "caught up" with the constraint), in the original model labor supply remained above the unconstrained during the recovery while the relationship has now inverted; *While the labor constraint prevents a precipitous drop during the initial fall, it causes an overall lower level of labor supply during the recovery.* Now, this recovery is rather long and persistent and more shocks may occur during this period. The effect that we can observe in a countinous simulation is therefore limited to the immediate effect when the shock occurs. Hence, the most important effect of the policy is the sharp increase in labor supply. This last piece of the puzzle is clearly visible in figure 8 where we have shown the variables' evolution through time with their actual levels. Note here that while in the original model the level of capital was typically lower in with the labor-constraint regime than in the base case, this is no longer the case: while capital is at similar values during positive transitions, it is greater



(a) Positive Shock; Capital (b) Positive Shock; Labor

Figure 8: Comparison of Transition Dynamics

during lower transition, pushing up labor as well. As a consequence productions is overall greater in the economy, leading to higher consumption. From this follows an important implication. While in general the labor-constraint policy seemed to have the desired effect during a down-turn in the original model, one of the consequences was to suppress capital growth during positive transitions. Now, on the contrary, this side-effect does not exist any longer and through greater volatility in labor the policy-regime is more efficient in its work. The final step now is to assess whether the policy may be welfare enhancing or not. In order to do so, we will use the same Monte Carlo Simulation method used before. For the sake of completeness, we have also computed the variance of relevant variables using Monte-Carlo approximation of the expectancy operator; $\text{Var}(Y) = \sum_{j=1}^{X} \left[\sum_{t=1}^{T} (Y_t(j) - \mu_j)^2 / (T-1) \right] / X$, where Y is the objective variable and $Y(j)$ is a generated sequence with mean μ_j. As before, we set a time-horizon of $T = 500$ and 5000 iterations. The results are reported in table 6. As can be seen, constraining labor is still not a welfare enhancing policy although the differences are very minute. The minimum wage, on the other hand, is gaudily not welfare enhancing. The reasons are probably related to the increased consumption's volatility. In fact, even is it reduces swings in capital stock, wage rigidity has very strong and direct negative effects on the demand of labor. However, some caution should be taken when evaluating these policies with this models, as we only consider one type of agent. With heterogeneous agents, some more dependent on labor income that others, this policy may be welfare enhancing for the intended sub-population. For instance, we have seen that the policy is highly efficient as a volatility-reducing tool, reducing swings for both capital and labor, and as a consequence for consumption as well.

Model	Divisible			Indivisible		
Policy	P.1	Free	P.2	P.1	Free	P.2
Exp. Utility	-0.65873	-0.65868	-0.6657	-36.3877	-36.3859	-36.3903
St. Deviation						
Capital	0.3430	0.3550	0.2393	0.4071	0.4656	0.3803
Labor	0.0016	0.0018	0.0039	0.0051	0.0077	0.0086
Consumption	0.0204	0.0205	0.0257	0.0243	0.0254	0.0310

Table 6: Comparison of Expected Lifetime Utilities

Further Possibilities

While the models presented in this paper are simple, they lend themselves to extensions in a variety of direction. First and perhaps most obvious, is to allow for a more forward looking production market where firms anticipate that their hiring policy in period t affects their manoeuvre-space in $t+1$. This extension can be easily implemented by converting the firm's maximization problem to an infinite horizon problem. In this way, we would have to find its value function and, assuming the Second Welfare Theorem applies, import its FOC to the consumer problem. The complication arises when one has to consider how the choice of ℓ_t affects the value function in $t+1$, which we have ignored in this paper since firms do not have any foresight. But with such foresight, one must specify what the functional form is and hence add another dimensionality to the problem. On the other hand, the optimal solution when the constraint is binding would still be the minimum choice of labor, at which point such added dimensionality adds little of value.

Another interesting extension that we have hinted at is to study how various types of households benefit from job-security policies. Perhaps the most important up-shot of this paper is that capital is fundamentally affected by labor-market policies (in our models) and so one should consider the behavior of those who's income is primarily dependent on capital. Moreover, given that the policy's intended target is the less-endowed, this would also add a dimension with respect to them.

Extensions in the policy directions are as many as imagination can conjure, where some are more sound and relevant than others. A fairly straightforward extension that we had as ambition to include was that of a "firing cost function" which could be thought of as a policy induced added cost. However this problem is not trivial, as it is not at all obvious what such a function should look like. Moreover, if it never admits profits, one must take great care in modeling the product market, lest no firms enter. In the end the task swelled to a proportion beyond a simple extension, but nevertheless it should be an interesting exercise to study how such a policy fares in comparison to the two studied here.

Concluding Remarks

In this paper we have analyzed the impact two typical job-protection policies may have on the economy. We have developed a simple model of general equilibrium assuming perfectly rational agents in a perfectly competitive market. The results we have found are very interesting and they open challenging new questions to explore with further study. The first result we found is related to the central importance of labor in the economy. In fact, we found that the minimum wage policy, through the constraint in the competitive definition of work compensation, increased incredibly the volatility of labor. As a consequence, production and consumption volatilities increase, with a negative impact on the agents' life-time utility. On the contrary, the labor-constraint policy, as it directly intervened on the labor variable limiting sharp reduction, lead to higher employment levels, slightly lower compensations but higher and more stable consumption over the various simulation periods. The policy revealed not to be welfare enhancing, though, due to the higher level of work (which enters with a negative sign into the utility function). However, this results open the door to a new series of analysis which could consider different types of agents, with different characteristics and utility functions, in order to verify if this kind of policy, which primarily aims to protecting the weakest portion of population, may be really welfare enhancing for its target population sub-segment.

References

Acemoglu, Daron (2009). *Introduction to Modern Economic Growth.* Princeton, N.J: Princeton University Press.

Flodén, Martin (2008). "A note on the accuracy of Markov-chain approximations to highly persistent AR(1) processes". In: *Economics Letters* 99.3, pp. 516 –520.

Hansen, Gary D. (1985). "Indivisible labor and the business cycle". In: *Journal of Monetary Economics* 16.3, pp. 309–327.

McCandless, George (2008). *The ABCs of RBCs.* Cambridge, Massachusetts and London, England: Harvard University Press.

Tauchen, George (1986). "Finite state markov-chain approximations to univariate and vector autoregressions". In: *Economics Letters* 20.2, pp. 177–181.

Appendix A

We begin with the problem in equation (IV). Define the value function as;

$$V(k_t, \ell_{t-1}, A_t) \equiv \max_{\substack{\{k_{t+1+s}\}_{t=0}^{\infty} \in \mathbb{R}_+^{\infty} \\ \{\ell_{t+s}\}_{t=0}^{\infty} \in \mathbb{L}_t^{\infty}}} E_0 \left\{ \sum_{s=0}^{\infty} \beta^s u(k_{t+s}, k_{t+1+s}, \ell_{t+s}, A_{t+1} | \Theta) \right\}. \tag{A.1}$$

Now, since the arguments of $V(\cdot)$ where arbitrarily chosen, this holds for any set of initial values. Since for all t, $k_t \in \mathbb{R}_+$, $\tilde{\ell}_t \in [0,1]$, $\ell_t \in [\tilde{\ell}_t, 1]$, both the control- and state space are non-empty convex sets. As for the goal-function, these controls map the goal function into the a convex set in $\mathbb{R}$. It's strict concavity and differentiability on $\mathbb{R}_- \cup \mathbb{R}_+$ ensures the existence of the maximum of $V(\cdot)$ on the ranges our parametrization give rise to. For further details see Acemoglu (2009, Ch. 15). Now, to derive the value function in (V), fix some time t and initial states k_t, $\tilde{\ell}_t$, and A_t. Then

$$\begin{aligned} V(k_t, \tilde{\ell}_t, A_t)_{A_t \in \mathcal{A}} &\equiv \max_{\substack{\{k_{t+1+s}\}_{t=0}^{\infty} \in \mathbb{R}_+^{\infty} \\ \{\ell_{t+s}\}_{t=0}^{\infty} \in \mathbb{L}_t^{\infty}}} E_0 \left\{ \sum_{s=0}^{\infty} \beta^s u(k_{t+s}, k_{t+1+s}, h_{t+s}, A_{t+s} | \theta) \right\}, \ A_t = A_i \\ &= \max_{\substack{\{k_{t+1+s}\}_{t=0}^{\infty} \in \mathbb{R}_+^{\infty} \\ \{\ell_{t+s}\}_{t=0}^{\infty} \in \mathbb{L}_t^{\infty}}} \Bigg\{ u(k_t, k_{t+1}, \ell_t, A_t) + \\ &\qquad \beta E_0 \sum_{s=0}^{\infty} \beta^s u(k_{t+1+s}, k_{t+2+s}, h_{t+1+s}, A_{t+1} | \theta) \Bigg\} \\ &= \max_{\substack{k_{t+1} \in \mathbb{R}_+ \\ \ell_t \in \mathbb{L}_t}} \{ u(k_t, k_{t+1}, \ell_t, A_t) + \beta V(k_{t+1}, A_{t+1} | \theta) \} \\ &= \max_{\substack{k_{t+1} \in \mathbb{R}_+ \\ \ell_t \in \mathbb{L}_t}} \left\{ u(k_t, k_{t+1}, \ell_t, A_t) + \beta \left(\sum_{j \in \mathcal{A}} p_{i,j} V(k_{t+1}, A_j) \right) \right\}. \end{aligned} \tag{A.2}$$

Since the initial states were arbitrary, this holds for all feasible pairs.

Appendix B

Consider some discrete interval of feasible capital stocks, $\mathcal{K} = \{k_{min}, k_2, k_3, ..., k_{N-1}, k_{max}\}$. For each of these, we wish to find the optimal control pair of k_{t+1}, ℓ_t, conditional on initial state A_i. While not necessary in principle, we restrict $k_{t+1} \in (x_-, x^+)$ for some lower and upper limit to speed up the algorithm. Recall that $\ell_t \in (\tilde{\ell}_t, 1) = \ell_t$. The economy in (V) has three state variables, k_t, $\tilde{\ell}_t$ and A_t. Thus, we must solve the Value Function for each possible combination. This is where the issue of dimensionality rapidly makes itself known as the computing cost increases exponentially as the set of each state variable increases. As for $\tilde{\ell}$, we construct a an ordered grid of $\boldsymbol{h} = \{h_0^{min}, h_0^2, .., h_0^{s-1}, h_0^{max}\}$, with $h_0 = \tilde{\ell})$ being possible lower bounds. Fix an h_0^s and make an initial guess $v_j^0(i) = \mathcal{K}(i)$ for each $k_t = \mathcal{K}(i)$ and for each $A_i \in \mathcal{A}$. Let the matrix of initial guesses per possible lower bound be V_j^0. Given the initial guess, for each initial state i solve;

$$V_j^1 = T(V_j^0) = \max_{\substack{\ell_t \in (h_0, 1) \\ k_{t+1} \in (x^-, x^+)}} u(k_t^i, k_{t+1}, \ell_t, A_j) + \beta \left(\sum_{q \in \mathcal{A}} pi, qV_q^0 \right), \ {}_{\substack{k_t \in \mathcal{K} \\ s \in \boldsymbol{h}}}.$$

Repeat for each case $h_0^s \in \boldsymbol{h}$. To complete the algorithm, define an approximation tolerance ζ and iterate until $|V_j^{s+1} - V^s| < \zeta$ for each case $s \in \boldsymbol{h}$. In general;

$$V_j^{r+1} = T(V_j^r) = \max_{\substack{\ell_t \in (h_0^s, 1) \\ k_{t+1} \in (x^-, x^+)}} u(k_t^i, k_{t+1}, \ell_t, A_j) + \beta \left(\sum_{q \in \mathcal{A}} p_{i,q} V_q^r \right), \ {}_{\substack{i \in \mathcal{K} \\ j \in \mathbb{N}^n \\ r \in \mathbb{N}_0 \\ s \in \boldsymbol{h}}}. \tag{B.1}$$

Which converge on the vector of fixed points as $r \to \infty$.

Appendix C

Set initial state $A_0 = A_i$. Then each $A_j \in \mathcal{A}$ occurs with probability $p_{i,j}$ by (1). These probabilities partition the probability space $\Omega = [0, 1]$. Thus, in a repeated draw from Ω each A_j occurs with a relative frequency of $p_{i,j}$. Ordering the ith row of Θ cumulatively, the sum of $\sum_1^j p_{i,j}$ act as an upper bound on the measure of the occurrence of each A_j, while the sum $\sum_1^{j-1} p_{i,j}$ act as the lower bound. Applying this schema for each j we partition Ω according to the measure of the relative frequency of each A_j conditional on $A_t = A_i$. Now, by drawing a random number (from the uniform distribution) and comparing it to the partitioning, we can determine A_{t+1};

Step 1: Determine state of A_t.

Step 2: Conditional on $A_t = A_i$, define the vector $P = [p_{i,j}{}_{j \in \mathcal{A}}$ as the i-conditional transition matrix. Order P in the inverse cumulative order[9]. Now define the partitioning by letting $\vartheta(j) = \sum_1^j p_{i,j} - \sum_1^{j-1} p_{i,j}$.

Step 3: Draw a random number p from the uniform distribution

Step 4: Iteratively apply the logical rule "if $p \leq \vartheta(j)$ then $A_{t+1} = A_j$". If not satisfied, repeat with $j = j + 1$ until the rule is satisfied.

Step 5: Repeat steps 1-4 for the full sequence of time T.

[9]Working with the cumulative order is also possible, but the inverse cumulative order is simpler to implement.